SZENT ISTVÁN EGYETEM

DESCRIPTION OF THE HEALTHY HUMAN KNEE JOINT KINEMATICS BASED ON EXPERIMENTS

Thesis of PhD work

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CONTENTS

NOTATION

$\boldsymbol{A}_t$	The transformation matrix of the tibial tracker's coordinate system in the absolute coordinate system
$\boldsymbol{A}_f$	The transformation matrix of the femoral tracker's coordinate system in the absolute coordinate system
$\boldsymbol{A}_{tátv}$	The transformation matrix of the tibial reference coordinate system in the absolute coordinate system
$\boldsymbol{A}_{fátv}$	The transformation matrix of the femoral reference coordinate system in the absolute coordinate system
$\boldsymbol{C}_{t,tanat}$	The transformation matrix of the tibial anatomical coordinate system in the tibial reference coordinate system
$\boldsymbol{C}_{f,fanat}$	The transformation matrix of the femoral anatomical coordinate system in the femoral reference coordinate system
$\boldsymbol{A}_{tanat}$	The transformation matrix of the tibial anatomical coordinate system in the absolute coordinate system
$\boldsymbol{A}_{fanat}$	The transformation matrix of the femoral anatomical coordinate system in the absolute coordinate system
$\boldsymbol{B}_{t,tanat}$	The transformation matrix of the tibial anatomical coordinate system in tibial tracker's coordinate system
$\boldsymbol{B}_{f,fanat}$	The transformation matrix of the femoral anatomical coordinate system in femoral tracker's coordinate system
$\boldsymbol{B}_{f,tjel}$	The transformation matrix of the tibial tracker's coordinate system in femoral tracker's coordinate system
$\boldsymbol{B}_{f,tanat}$	The transformation matrix of the tibial anatomical coordinate system in femoral tracker's coordinate system
$\boldsymbol{D}_{f,tanat}$	The transformation matrix of the tibial anatomical coordinate system in femoral anatomical coordinate system
φ	Angle of flexion-extension
γ	Angle of adduction-abduction
ρ	Angle of rotation
a_1, a_2, a_3	The slope of the trilinear function on the first, second and third section
φ_1	The limit of the screw-home motion (first and second section)
φ_2	The limit of the second and third section

ρ_0	The axial section of the trilinear function
ρ_{10}	The rotation at the limit of the first and second section
ρ_{20}	The rotation at the limit of the second and third section
$a_{1,C}, a_{2,C}, a_{3,C}$	The slope of the reference function on the first, second and third section
$\rho_{10,C}$	The rotation at the limit of the first and second section of the reference function
$\rho_{20,C}$	The rotation at the limit of the second and third section of the reference function
fh	The center of caput femoris
mepi, *lepi*	medial and lateral epicondyle
hf	head of fibula
tt	tibial tuberosity
mm, *lm*	medial and lateral malleolus
j	The ID of the measurement data points
l	The total number of the measurement data points
n_1, n_2, n_3	The number of the measurement data points on the first, second and third section of the trilinear function
p	The ID of the cadaver subject
k	The ID of the measurement data series of the cadaver subject
$s_1, s_2, s_3, s_{ö}$	The standard deviation of the trilinear and the reference function on the first, second and third section and the global standard deviation of the functions

1. INTRODUCTION, GOALS

Today, it is possible to investigate any kind of motions with the use of computers and positioning systems, therefore the study of human movements has become widely and thoroughly researched. One of the most important articles is understanding the knee joint movements.

The most important motion describing quantities are the rotation and flexion-extension. The rotation can be described as the function of flexion-extension. My goal is to determine the healthy human knee joint's kinematical parameters, the correlations between them and the describing mathematical model based on in vitro experiments, which is a fundamental research topic. Currently, such mathematical model is not available in the literature.

The knee joint is one of the largest and most complex joints of the human body, but it is also quite vulnerable. Over time, knee prosthesis implantation may be necessary, to which the proper prosthesis design is essential. Naturally, it is needed to know how the healthy knee joint really works.

The longevity can be the goal during prosthesis design. The attrition of the prosthesis is caused by the wear while loosening is caused by the improper movement conditions. To ensure this, a mechanical analysis of the knee was carried out by defining a new mathematical model. By the use of this model the movement of the prosthesis approximates the healthy knee joint movement. The large numbers of the prosthesis implantation shows the actuality of the research.

One of my goals was to develop an experimental method for creating the mathematical model. The necessary measurements can be performed on a verified experimental apparatus using cadaver knee joints and the motion parameters can be determined using this method. It is necessary to determine whether the knee joint movements and the investigations are affected by which parameters mostly. The experimental apparatus must be designed and the measurements must be carried out accordingly.

The evaluation method of the experiments should be developed also. After that, my main goal is to create a function, with which the healthy human knee joint movement is basically characterized.

This result can be used as a fundament for knee prosthesis design and their comparative testing.

2. MATERIAL AND METHOD

The conditionalities to the experimental apparatus needed to the experiments, the experimental apparatus built on the basis of these and the experimental protocol are described in this chapter.

2.1. Experimental model and apparatus

The experiments were based on the statements also described in the literature that the motions of the knee joint are determined by the surfaces of the joint surfaces. Cadaver knee was chosen as experimental model to the measurements, it is suitable for the experiments due to the same geometry.

Fig. 2.1: Experimental model

2.1.1. The cadaver knee, as an experimental model

The geometry of the cadaver knee (Fig. 2.1) corresponds to the geometry of the living human knee joint, the ligaments are also present in the cadaver model, the ethical rules do not affect to the used techniques during the experiments. It must be noted that there is no muscle tone in the cadaver knee joint compared to the the living body, so it had to be modelled.
The motions of the cadaver knee are tested only in some movement mode with the experimental apparatus detailed in section 2.1.3. These are

- the flexion of the horizontally held feet by ever-increasing force (type 1)
- the flexion of the horizontally held feet by its self weight (type 2)

The quadriceps femoris is the only modelled muscle, which is the only acting muscle during the flexion of the horizontally held knee, in my examinations. The muscle was modelled in the experimental model with a rubber muscle model. Its characteristic is linear in between the applied load limits. It is an adequate model for modelling the muscle force and its changing during the living human knee flexion. The applied spring constant of the rubber is: 1.9 N / mm.

2.1.2. The subjects of the experiments

The experiments were carried out on ten cadaver joints (6 right, 4 left knee) of six men (age: 40-68 years, mean age: 54.5 years) subjects, which were preceded by standard physical examination of the subjects by a doctor. The condition of the joints were also examined by a doctor after carrying out the experiments and the joints were not observed for any signs of deterioration. The left joint of subject 5 (P09J) had to be excluded from the evaluation of the experiments due to gross error. The problem is caused by the Polaris malfunction. In addition, the measurement type 1 carried out on right joint of subject 5 (P09J) joint had to be also excluded from the evaluation. In this case the movement did not start from completely extended position.

2.1.3. Criteria for development of experimental apparatus

An experimental apparatus had to be built which allows the free motions of parts of the knee occurring during the movement. The other five kinematic parameters are created as a function of forced flexion-extension as the effect of cartilage geometry at the examined motion form. These movements have to be enabled by the apparatus. The movement causing loads should not prevent the motions of the knee driven by the cartilages. The aim of the apparatus design and creation was to be carried out experiments on cadaver knees which provide kinematic data. The muscles of the cadaver knees do not work, so they need to be modelled. The first step of the apparatus design was the modelling of the acting muscles. It may be important to know the contact surfaces between the condyles during the analysis of the joint kinematics. These can be defined by MRI or CT devices. Thus, the apparatus should be made of non-magnetic material with geometric dimensions which makes it suitable for carrying out these measurements. The apparatus had to ensure long-term, permanent provision of discrete positions of the knee and continuous movement also.

2.1.4. Description of the apparatus

The muscle model (Fig. 2.2.) is connected with one end to the experimental model, while with the other end to the body of the apparatus made of wood through a screw. The position of the knee and the tension force in the muscle model can be adjusted and the moving of the tibia can be solved by this screw. The joint is connected to the apparatus via intramedullar textile bakelite rods.

The rod cemented into the femur is mounted to the base plate of the apparatus body. The clamped femur is positioned horizontally. A moving plate is mounted to the rod cemented into the tibia perpendicular to the rod. The load is represented by the self-weight which is connected to experimental model through a ball joint placed on the axis of the tibia. The free movement of the knee joint is ensured. The knee can be loaded in two ways until required flexion and extension.

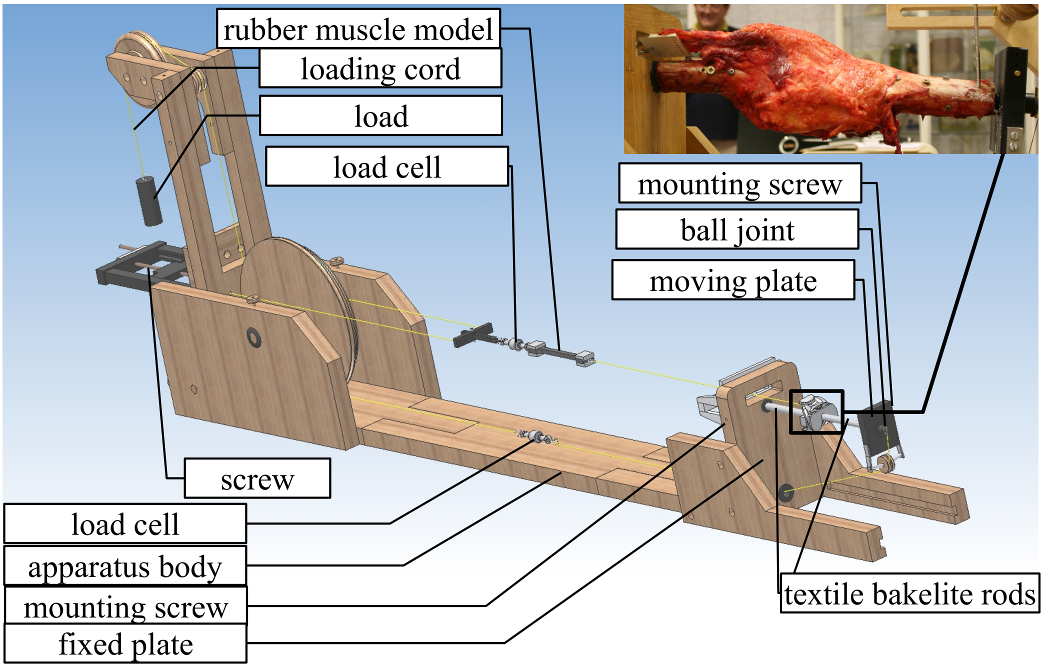


Fig. 2.2: The 3D model of the experimental apparatus

The moving of the knee is ensured by a constant mass flow water spray poured in and out to a bowl connected to the load cord in measurement type 1.
The moving of the knee is ensured by decreasing and increasing of the muscle model force by a screw from its stretched position in measurement type 2. The joint was moved three times in each case.

2.1.4.1. The measuring system

Changes of the position of the tibia were determined by Polaris (Northern Digital Inc., Waterloo, Ontario, Canada) optical positioning measurement system (Fig. 2.3) relative to the stretched position of the knee (the validation error of the Polaris is 0.35 mm according to the manufacturer) – after validation described in subsection 2.1.4.2. The moving trackers of the system were mounted rigidly to

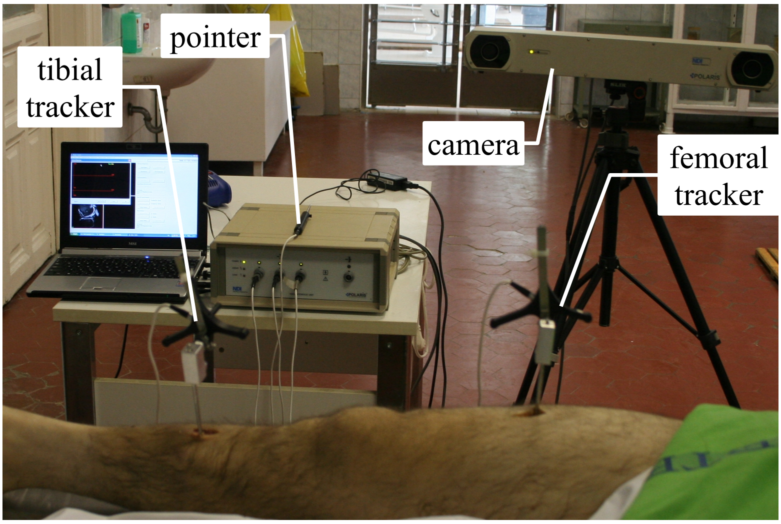


Fig. 2.3: The Polaris measuring system before mounting on the apparatus

the fixed femur and the moving tibia. The angles which describe the position of the tibia in the VAKHUM joint coordinate system can be calculated from the data measured by the Polaris which describe the position of the bones in the Polaris coordinate system. The load was measured by a load cell (HBM U9B 0.5 kN) connected to the load cord (Fig. 2.2) and the muscle force was measured by a load cell (HBM U9B 0.5 kN) connected to the muscle model. The measured force values were recorded and evaluated by a Spider 8 (Hottinger Baldwin Messtechnik GmbH, Darmstadt, Germany) data acquisition system.

2.1.4.2. The validation of the apparatus

A cardan joint was used for modelling the cadaver knee joint during validation of the apparatus (Fig. 2.4). The cardan joint is suitable for validation, because the axes of the cardan joint are the same as the axes of the joint coordinate system used by me due to its design. The angles describing the position can be adjusted along the axes of the cardan joint.

Fig. 2.4: Validating cardan joint

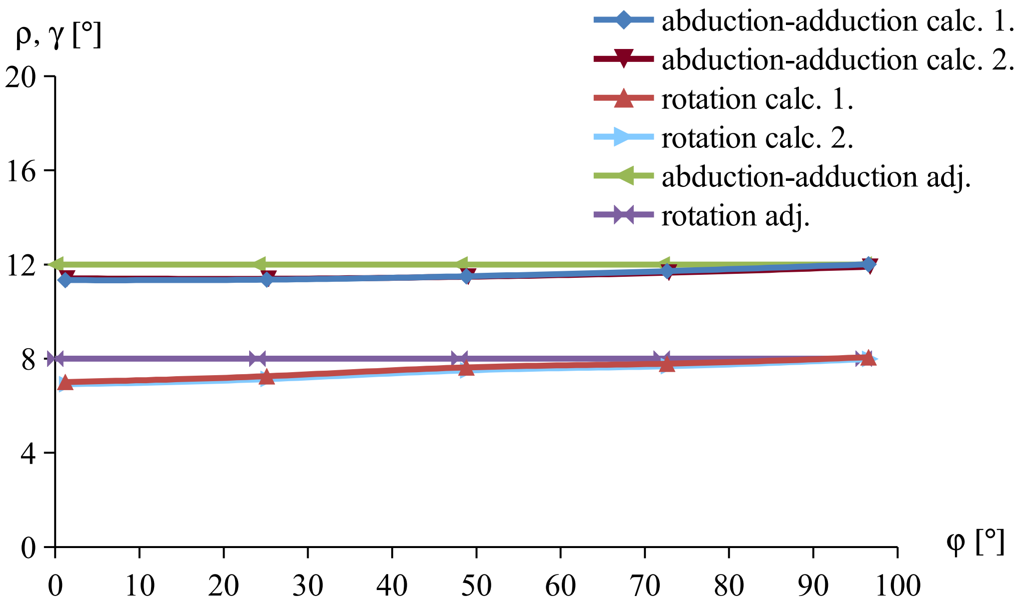


Fig. 2.5: The measurement system's verifing curves and repeatability error (φ – flexion-extension; γ – abduction-adduction; ρ – rotation)

The Polaris system was placed as prescribed by the manufacturer. Measurement was performed by the Polaris after setting the flexion-extension, abduction-adductio and rotation angles along the axes of the cardan joint. The measurements were performed two times to each adjustment in order to determine the repeatability error. The kinematic parameters were calculated from measurement data of the Polaris. The calibration curve and repeatability error can be seen in Fig. 2.5. in case of an adjustment.

2.2. The measurement protocol for creating the anatomical coordinate system

The knowledge of the placement of the anatomical landmarks is required to use the VAKHUM type anatomical coordinate system. The experimental protocol was defined in such a way that it is possible to measure then transform them to resected joint. The experiments were carried out on resected joints, which were evaluated in this coordinate system. In summary the steps of the measurements are the next:

- Loosening of rigor mortis
- Screwing threaded rods into the femur and the tibia
- Mounting the Polaris trackers to the threaded rods
- Assembling the Polaris at the factory guide
- Moving the leg circular during continuous measurements to determine the caput femoris (spherical piece) center (the center of the spherical motion)
- Dissection of the thigh and lower leg and fixing markers (6-6 pieces) into the femur and the tibia (possibility of the transformation which is needed because of resection)
- Measuring the position of the other anatomical landmarks an markers with using the pointer of the Polaris (discrete measurements)
- Resecting the knee joint, removing the threaded rods
- Sewing the load clamp to the ligament of m. quadriceps femoris
- Cementing the intramedullar textile bakelite rods into the bones
- Re-screwing threaded rods into the femur and the tibia
- Fixing the knee joint into the apparatus, connecting the rubber muscle model to the load clamp
- Mounting the trackers of the Polaris to threaded rods
- Assembling the Polaris at the factory guide
- Setting the knee joint into default position (anatomical extended position) with using the muscle model force and the load
- Measuring the position of the anatomical landmarks an markers with using the pointer of the Polaris (discrete measurements)
- Carrying out measurements (continuous measurements, three times repetition)

- Dissection of the joint, searching for abnormalities, 3D scanning of the bones

The measured data are:

- The position of the following anatomical landmarks: medial and lateral malleolus, tibial tuberosity, caput fibulae, medial and lateral epicondyle,
- the position of the markers,
- to the determination of caput femoris center: changes of the position of the femoral Polaris tracker,
- to the determination of kinematical parameters: changes of the position of the coordinate system connected to the femoral and tibial trackers during the continuous measurements.

3. RESULTS

In this chapter, I present my results, the trilinear functions fitted to the measurement data, and then I describe the objective function which can be defined on the basis of experiments.

3.1. The steps of the measurement data evaluation

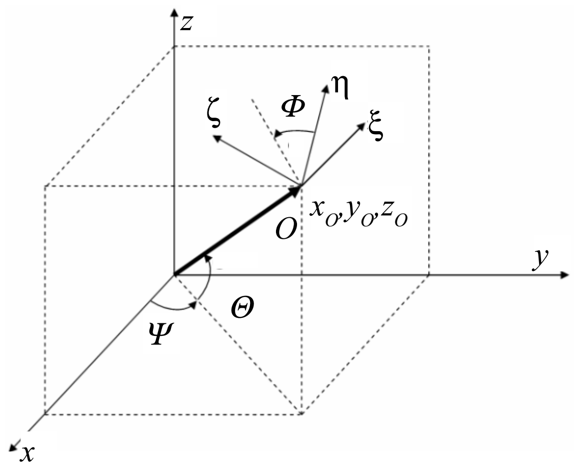


Fig. 3.1: Definition of the position describing data of the Polaris (x,y,z – absolute coordinate system; ξ,η,ζ – tracker's coordinate system; O – origin of tracker's coordinate system, x_O,y_O,z_O – the coordinates of the origin in the absolute coordinate system; Ψ,Θ,Φ – the position of tracker's coordinate system describing Euler-type angles)

The raw data are known during the experiments in the absolute coordinate system fixed to the camera of the Polaris (the definition of these can be seen in Fig. 3.1). The angles – which describe the motion of the tibia – have to be determined from these raw data with mathematical methods. The method, developed by me consists of two main steps:

- The determination of the anatomical coordinate systems:
  - At first, the caput femoris center (fh) (the center of the spherical motion) is determined from the continuous measurement data of the Polaris femoral tracker which are recorded by the Polaris during the circular motion of the leg.

- In the next step, reference coordinate systems, connected to the femur and tibia have to be determined relative to the absolute coordinate system with theirtransformation matrices $(\mathbf{A}_{tátv}, \mathbf{A}_{fátv})$. These coordinate systems are connected to the markers fixed in the bones. The use of them is appropriate because anatomical landmarks are removed during the resection which are necessary to the definition of the anatomical coordinate systems. The anatomical coordinate systems are inadequate without these reference coordinate systems.
- In the end, the femoral and tibial anatomical coordinate systems have to be determined relative to the reference coordinate systems with their transformation matrices $(\mathbf{C}_{t,tanat}, \mathbf{C}_{f,fanat})$.

– The evaluation of the measurement data:
  - The transformation matrices of the reference coordinate systems $(\mathbf{A}_{tátv,j}, \mathbf{A}_{fátv,j})$ and the tracker's coordinate systems $(\mathbf{A}_{t,j}, \mathbf{A}_{f,j})$ have to be determined again in the absolute coordinate system in every measured position because of the resection.
  - The transformation matrices of the anatomical coordinate systems relative to the absolute can be calculated with the following formulas:

$$\mathbf{A}_{tanat,j} = \mathbf{C}_{t,tanat} \mathbf{A}_{tátv,j} ; \mathbf{A}_{fanat,j} = \mathbf{C}_{f,fanat} \mathbf{A}_{fétv,j} .$$

  - The transformation matrices of the anatomical coordinate systems relative to the trackers are:

$$\mathbf{B}_{t,tanat,j} = \mathbf{A}_{tanat,j} \mathbf{A}_{t,j}^{-1} ; \mathbf{B}_{f,fanat,j} = \mathbf{A}_{fanat,j} \mathbf{A}_{f,j}^{-1} .$$

  - The transformation matrix of the tibial tracker relative to the femoral tracker is:

$$\mathbf{B}_{f,tjel,j} = \mathbf{A}_{t,j} \mathbf{A}_{f,j}^{-1} .$$

  - The transformation matrix of tibial anatomical coordinate system relative to the femoral tracker is:

$$\mathbf{B}_{f,tanat,j} = \mathbf{B}_{t,tanat,j} \mathbf{B}_{f,tjel,j} .$$

  - In the end, the transformation matrix of the tibial anatomical coordinate system relative to the femoral anatomical coordinate system is (

$\mathbf{D}_{f,tanat,j}$ - in function of flexion-extension, abduction-abduction and rotation):

$$\mathbf{D}_{f,tanat,j} = \mathbf{B}_{f,tanat,j} \mathbf{B}_{f,fanat,j}^{-1} .$$

- The $\mathbf{D}_{f,tanat,j}$ transformation matrix can be written in the following form:

$$\mathbf{D}_{f,tanat,j} = \begin{bmatrix} e_{x_s x_t} & e_{x_s y_t} & e_{x_s z_t} \\ e_{y_s x_t} & e_{y_s y_t} & e_{y_s z_t} \\ e_{z_s x_t} & e_{z_s y_t} & e_{z_s z_t} \end{bmatrix} .$$

- The elements of the matrix can be calculated from the angle of flexion-extension (φ), adduction-abduction (γ) and rotation (ρ). The kinematical angles can be calculated from the known matrix (each angle is 0°, if the femoral and tibial anatomical coordinate systems are parallel). The flexion-extension, adduction-abduction and rotation are the following in this order:

$$\varphi = \arctan\left(-\frac{e_{y_s x_t}}{e_{y_s y_t}}\right) ; \; \gamma = \arcsin\left(e_{y_s z_t}\right) ; \; \rho = \arctan\left(-\frac{e_{x_s z_t}}{e_{z_s z_t}}\right) .$$

3.2. The results of the experiments

3.2.1. Parameters of the investigations

The significance of the adduction-abduction in the clinical practice is much less than that of the rotation, so this topic is out of scope of the thesis. Many parameters have effect on the rotation-flexion values. Therefore, my experiments were carried out using the following well-defined parameters:

- the experimental model is cadaver knee joint,
- the experimental method is described in Section 2.2,
- the evaluation method is as follows:
  - the used coordinate system to the evaluation: VAKHUM type,
  - the definition of the angles and axes: VAKHUM type,
  - the transformation method of the coordinate systems: as it is described above

- the examined motion type: flexion-extension of the stretched knee joint,
- the examined person has healthy knee joint,
- the source of measurement error is the positioning of the joint,
- the source of the measurement system error is the error of the Polaris.

3.2.2. Error causing factors

The present section explains how the determination of anatomical coordinate systems influences the rotation-flexion curves.

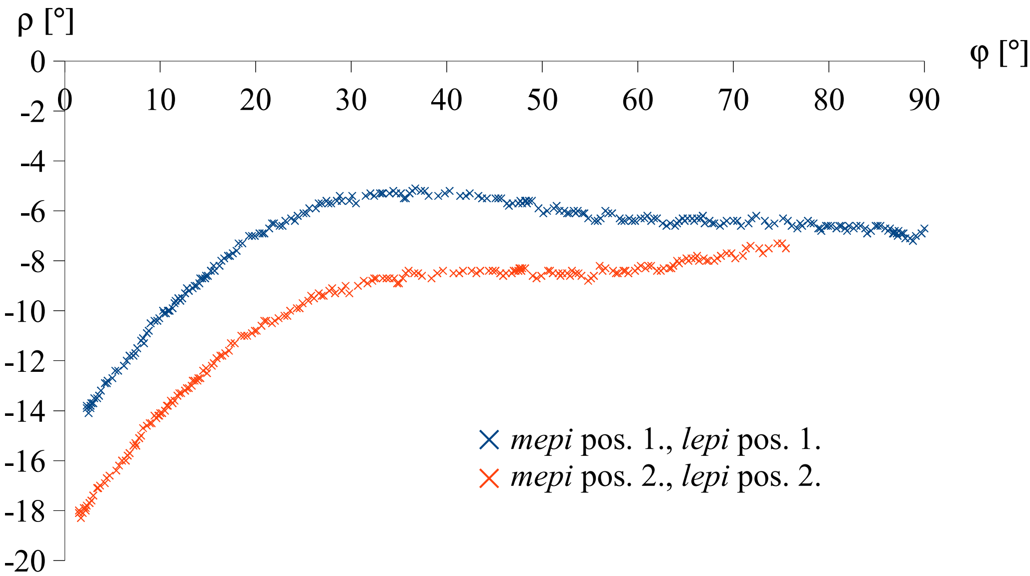


Fig. 3.2: The effects of the determination inaccuracy of the anatomical landmarks defining the femoral lateral axis to the rotation-flexion diagram based on two measured position (*mepi* – medial epicondyle; *lepi* – lateral epicondyle; p=5; k=4; φ – flexion-extension; ρ – rotation)

The accuracy of epicondylar points' determination (the error of femoral lateral axis' direction) has significant effect on the value of rotation. The angle error of this axis has significant effect on the character of the rotation-flexion curve also (Fig. 3.2). The reason of this is that the flexion-extension is the first Euler-type angle in the VAKHUM joint coordinate system and the flexion-extension's axis is determined by the femoral lateral axis. Other anatomical landmarks (e.g. FFC points) can be searched for increasing the accuracy instead of the epicondylar points. The measurement of these landmarks give more accuracy in case of same

number of repetitions. The original VAKHUM-type femoral anatomical coordinate system was modified by me. The landmarks, which can be determined more precisely were used to determine this coordinate system.
The medial and lateral malleolus are close to the origin of the tibial anatomical coordinate system, so the inaccuracy of these points' determination cause more error on the direction of the axis to which the rotation is calculated (Fig. 3.3). The inaccuracy of malleolus points' determination has great effect on the direction of axis. Due to the inaccuracy the rotation-flexion curves slide parallel.

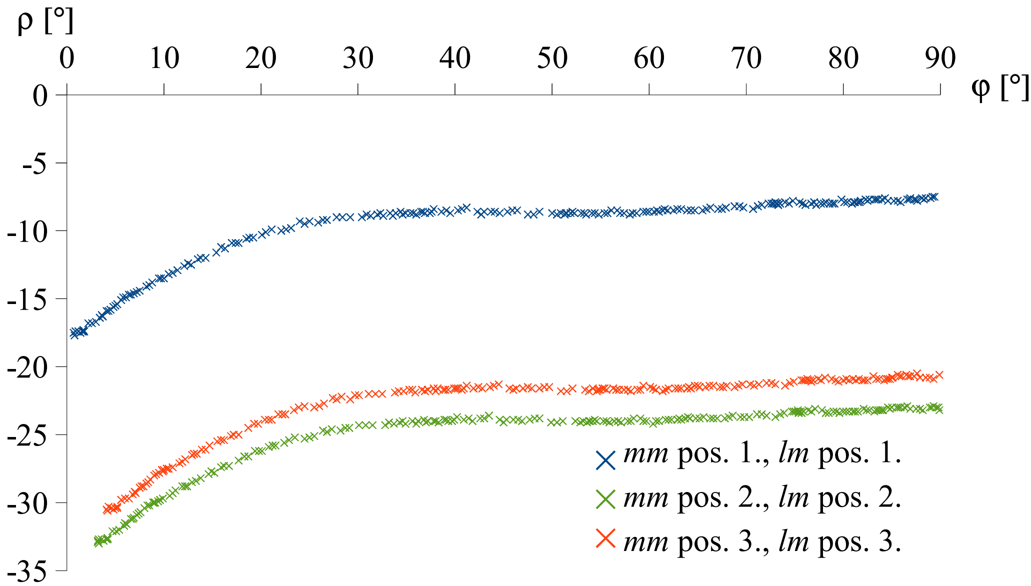


Fig. 3.3: The effects of the determination inaccuracy of the anatomical landmarks defining the tibial lateral axis to the rotation-flexion diagram based on three measured position (*mm* – medial malleolus; *lm* – lateral malleolus; p=5; k=4; φ – flexion-extension; ρ – rotation)

3.2.3. Results of the experiments

The results of the measurements carried out on ten cadaver knee joints can be seen on Fig. 3.4 calculated with the method developed by me (in case of parameters detailed in Section 3.2.1 and after excluding gross errors).

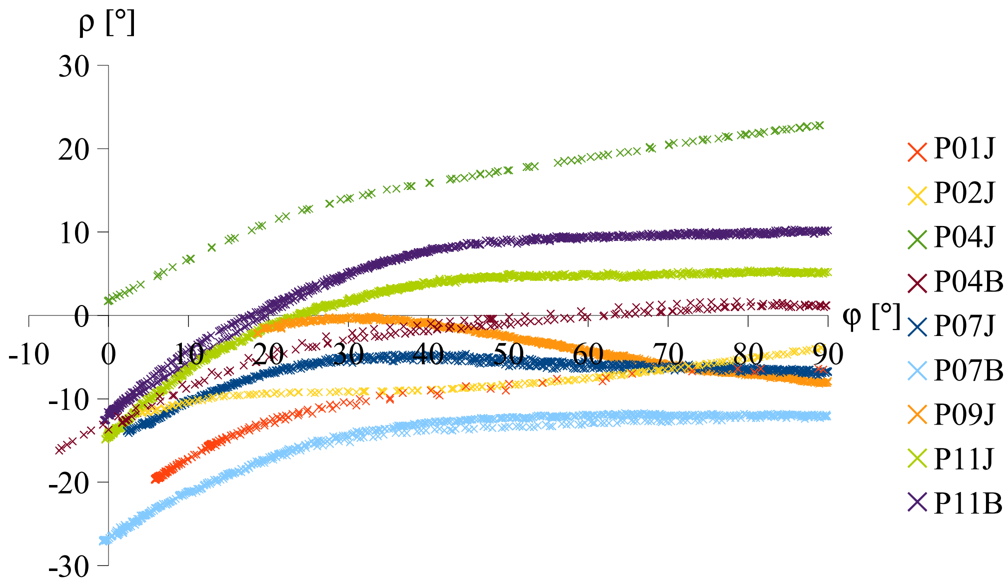


Fig. 3.4: The measured data of rotation – flexion (k=2; φ – flexion-extension; ρ – rotation)

The rotation-flexion curve can be slide into the origin due to the error-causing effect of the tibial lateral axis on the basis described in Section 3.2.2. The measurements are comparable due to this transformation. The method of the transformation is that the rotation value of the fitted trilinear function, which belongs to zero flexion, has to be involved out of the measurement data.

3.3. Creation of the reference function

The reference function is the mathematical model of the healthy human knee joint's rotation-flexion motion. The rotation-flexion function can be divided into three phases. There is a so-called screw-home motion in starting phase, a completely free motion in the ending phase of the flexion, and the phase between the other two, which can be considered a transitional phase (according to the literature also). So the rotation-flexion function can be modelled with a trilinear function.

3.3.1. The approximation of the kinematical diagrams with trilinear function

The starting phase of flexion is the screw-home motion which is the range of $0° - \varphi_1$ (Fig. 3.5). There is no consensus on the value of φ_1. The value of this

can be between 10° and 30°, according to the literature. The last phase of flexion is the free motion's phase, which starts from φ_2. The range $\varphi_1 - \varphi_2$ between the other two phases is a transitional phase. So the limits of the phases are $\varphi_0 = 0°; \varphi_1; \varphi_2$, and the total range of the fitting is $\varphi = 0 - 90°$.
The kinematical data are approximated with a linear function (in case of all measurement data set) on the phases appointed by these phase limits (3.1). The trilinear function is fitted to the measurement data on the phases of flexion on the basis of least-squares, considering the continuity.

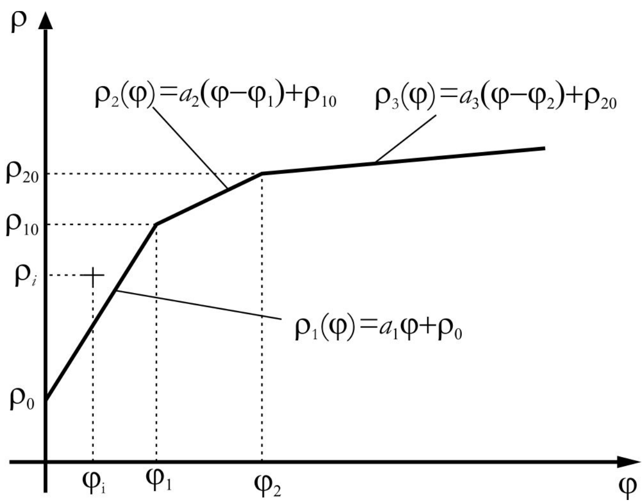


Fig. 3.5: The sketch of trilinear approximation

$$\begin{aligned} \rho_{1,pk}(\varphi) &= a_{1,pk}\,\varphi + \rho_{0,pk} \\ \rho_{2,pk}(\varphi) &= a_{2,pk}(\varphi - \varphi_1) + \rho_{10,pk} \\ \rho_{3,pk}(\varphi) &= a_{3,pk}(\varphi - \varphi_2) + \rho_{20,pk} \end{aligned} \tag{3.1}$$

3.3.2. Method for determining the limit of the screw-home motion

The concept of the determination is that the trilinear function has to be fitted to the measurement data on the whole range of flexion in such a way that the total standard deviation has to be smallest possible (3.2). The total standard deviation (3.2) is calculated in function of φ_1 and φ_2 phase limits. The flexion limits of the screw-home (φ_1) and free (φ_2) motions are at the minimum of this function

$$s_{ö,pk} = \sqrt{\frac{\Delta}{n_{3,pk} - 4}}, \tag{3.2}$$

where

$$\Delta=\sum_{j=1}^{n_{1,pk}}\left[\rho_{j,pk}-\left(a_{1,pk}\varphi_{j,pk}+\rho_{0,pk}\right)\right]^2$$
$$+\sum_{j=n_{1,pk}+1}^{n_{2,pk}}\left[\rho_{j,pk}-\left(a_{2,pk}\left(\varphi_{j,pk}-\varphi_1\right)+\rho_{10,pk}\right)\right]^2$$
$$+\sum_{j=n_{2,pk}+1}^{n_{3,pk}}\left[\rho_{j,pk}-\left(a_{3,pk}\left(\varphi_{j,pk}-\varphi_2\right)+\rho_{20,pk}\right)\right]^2.$$

The minimum of this function can be determined only with numerical method. The total standard deviation (3.2) has to be calculated in such a way that the φ_1 has to be changed between 10° and 30° range of flexion while the φ_2 has to be changed between 35° and 70° range of flexion. The φ_1, φ_2 data pair can be calculated with this method, which are the phase limits of the trilinear function.

The limits of screw-home and free motions

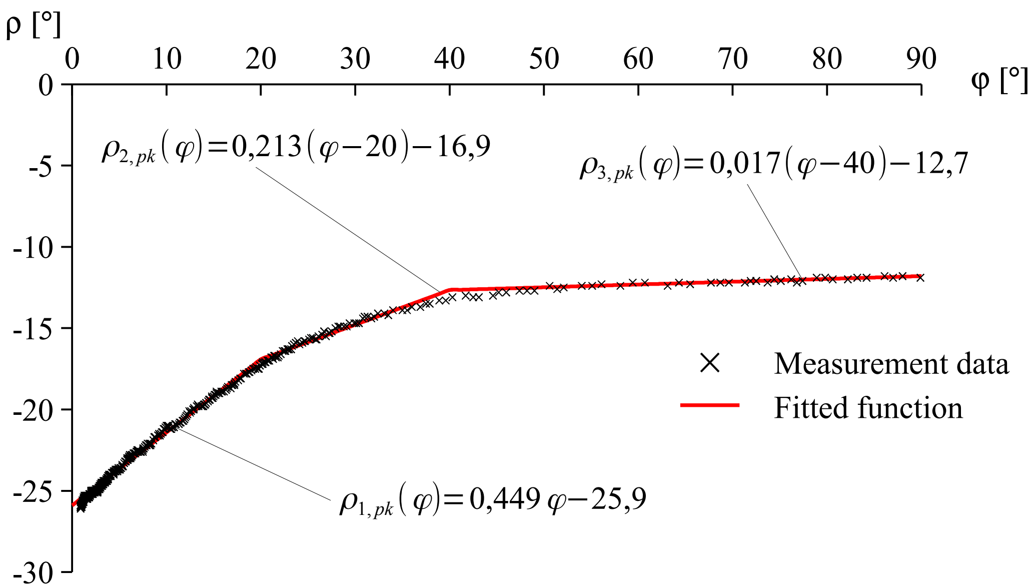


Fig. 3.6: Example for the trilinear approximation (p=6; k=1; φ – flexion-extension; ρ – rotation)

A trilinear function was fitted to the rotation data set determined from the measurement data (3.1). An example can be seen on Fig. 3.6. The function of the variance, using Formula 3.2 in function of φ_1 and φ_2 phase limits can be seen on Fig. 3.7.

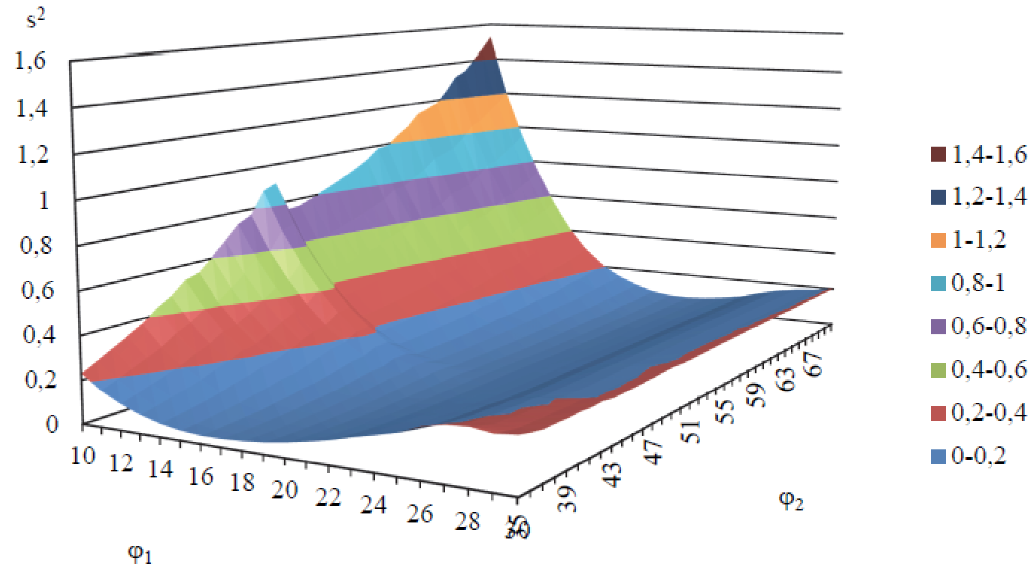


Fig. 3.7: Determination of the limits of sections based on the minimum of standard deviation (p=1, k=1, φ_1 – the range of flexion of the screw-home motion's limit determination, φ_2 – the range of flexion of the free motion's limit determination)

The gross error examination of φ_1 values was performed using F-test. The gross error examination of φ_2 values was not performed, because the filtering of gross errors was depended on φ_1 values. The average of screw-home motion limits, which belonged to the measurement data set was calculated after excluding gross errors. The standard deviation of the averages was calculated. The limit of the screw-home motion is with 95% probability:

$$\varphi_1 = \bar{\varphi}_1 \pm 2{,}5\, s_1 = 17{,}75 \pm 1{,}075\,^\circ .$$

The limit of screw-home motion is 20° of flexion after rounding considering the anatomical characteristics. The φ_2 limits have to be determined after this also. The limit of free motion is with 95% probability:

$$\varphi_2 = \bar{\varphi}_2 \pm 2{,}5\, s_2 = 42{,}28 \pm 4{,}8\,^\circ .$$

The limit of free motion is $\varphi_2 = 40\,°$ after rounding considering the anatomical characteristics.

3.3.3. The determination of the reference function

It was established after the evaluation of the measurement data sets that the rotation-flexion data can be approximated by trilinear function well. So the whole phenomenon (the average rotation-flexion function) can be approximated by trilinear function also.
The rotation-felxion data have to be slide parallel with $\rho_{0,pk}$ on the rotation axis. The goal of this transformation is that all of the measurement data start from the origin of the rotation-flexion coordinate system. The reason of the sliding is that the rotation is the third Euler-type angle in the flexion-extension, adduction-abduction, and rotation angles in the definition of angles of the VAKHUM joint coordinate system. Therefore, the value of flexion-extension and adduction-abduction angles do not change when the zero position of rotation is changed. Measurement data sets are resulted after sliding the measurement data. A trilinear function has to be fitted to the data set between the determined limits.
The kinematical data (all of the measurement data) are approximated with a linear function in the phases appointed by the limits determined earlier:

$$\begin{aligned}
&\rho_{1,kC}(\varphi) = a_{1,kC}\,\varphi\,, \\
&\rho_{2,kC}(\varphi) = a_{2,kC}(\varphi - \varphi_1) + \rho_{10,kC}\,, \\
&\rho_{3,kC}(\varphi) = a_{3,kC}(\varphi - \varphi_2) + \rho_{20,kC}\,.
\end{aligned} \tag{3.3}$$

The fitting has to be carried out on the all measurement data set on the principle of least squares.

The coefficients of reference function

The phase limits, which are needed to the trilinear approximation of rotation-flexion function were determined earlier. The limits are:

$$\varphi_0 = 0\,°;\ \varphi_1 = 20\,°;\ \varphi_2 = 40\,°\ .$$

Trilinear functions was fitted on the measurement data sets considering these phase limits. The coefficients of average trilinear functions (reference functions – 3.3) can be determined on the knowledge of these. These functions can be seen on the figure in Thesis 3, on page 24.

4. NEW SCIENTIFIC RESULTS

My new scientific results in the area of the biomechanics of healthy human knee joint focusing on the rotation-flexion motion are:

1. I defined a novel experimental method. I designed a multi-purpose calibrated experimental apparatus which uses the novel experimental method. This apparatus ensures experiments on cadaver knee joints and definition of the mechanical model of healthy human knee joint.

   The experimental apparatus is designed to be suitable for modelling the healthy human knee joint with cadaver knee joint. I created a model for the force system creating the motion. I selected the sizes and the material of the apparatus so that it is usable in CT and MRI equipments. I designed the apparatus suitable for discrete and continuous measurements also because of using it in these equipments. I modelled the m. quadriceps femoris with rubber muscle model in favour of the simulation of active knee motion. I created the movement without constraint loading through a cord fixed to the centerline of the rod cemented into the tibia. I validated the measurement system with a cardan joint which models the knee joint mounted into the apparatus. I determined the calibration curve and the repetition error of the system using the cardan joint. The position of the cadaver knee joint was measured in the apparatus with Polaris optical 3D positioning system.

2. I developed the evaluation method of the measurement data. The kinematical parameters (flexion-extension, adduction-abduction, rotation) – which describe the motion – can be calculated from the data measured by the Polaris (three distances and three angles measured in the absolute coordinate system attached to the camera) with multiple matrix operations.

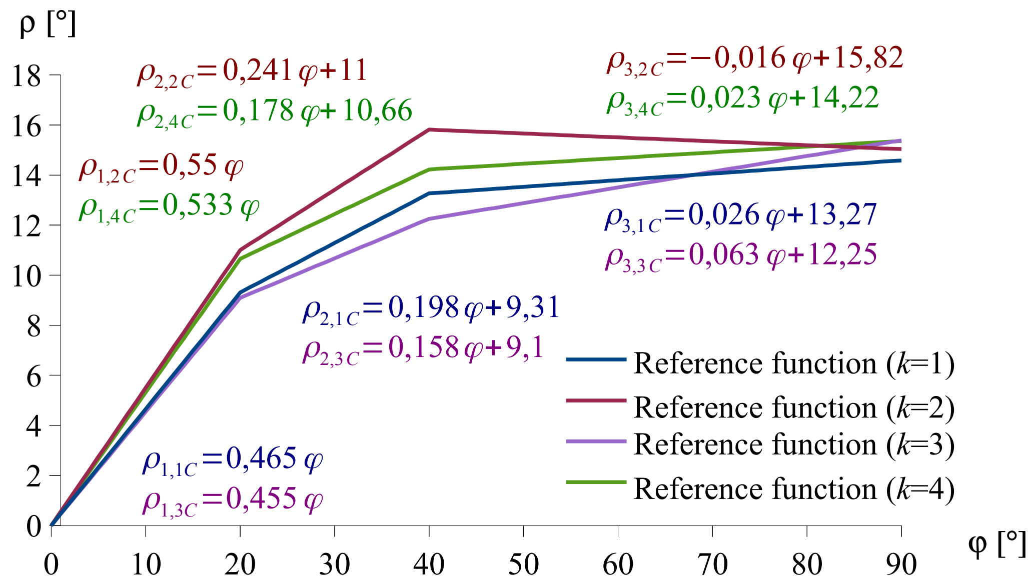
   I decomposed the evaluation method into two main steps. I determined the anatomical coordinate system needed to the evaluation in the first main step. At first I determined the caput femoris center from the measurement data of the Polaris' femoral tracker recorded during the circular motion of the leg. Then I determined the reference coordinate system attached to the markers fixed in the bones, which ensured the possibility of the using of the resected anatomical landmarks. Finally I determined the used anatomical coordinate systems relative to the reference coordinate system. In the second main step I defined the evaluation method of the measurements. At first I determine the transformation matrix with multiple matrix operations which describes the relative position of the anatomical coordinate systems:

$$\mathbf{D}_{f,tanat,j} = \begin{bmatrix} e_{x_s x_t} & e_{x_s y_t} & e_{x_s z_t} \\ e_{y_s x_t} & e_{y_s y_t} & e_{y_s z_t} \\ e_{z_s x_t} & e_{z_s y_t} & e_{z_s z_t} \end{bmatrix}.$$

Then I calculate from this matrix the angles which describe the position of the tibia relative to the femur (flexion-extension (φ), adduction-abduction (γ), rotation (ρ)):

$$\varphi = \arctan\left(-\frac{e_{y_s x_t}}{e_{y_s y_t}}\right);\ \gamma = \arcsin\left(e_{y_s z_t}\right);\ \rho = \arctan\left(-\frac{e_{x_s z_t}}{e_{z_s z_t}}\right).$$

3. I created the mathematical model of the rotation-flexion function of the healthy human knee joint and the evaluation method of that. The method is also usable for other joint modelling. I calculated the coefficients of reference function which describes the rotation-flexion motion of the healthy human knee joint.



Reference functions in case of different anatomical coordinate systems and measurement types (φ – flexion-extension; ρ – rotation)

I modelled the rotation-flexion function with a trilinear curve within a flexion range of $\varphi = 0 - 90°$, the flexion limits of the sections are the limit of screw-home motion (φ_1) and starting flexion value of the free motion (φ_2). I determined on the basis of the results of experiments that the rotation-flexion curves determined in the same joint but different anatomical coordinate system can be translated on the axis of rotation. I developed the transformation method with which the curves can be translated into the origin. I determined the reference function (which can be seen on the figure above) on the basis of the experiments and the above described statements. It describes the rotation-flexion motion of the healthy human knee joint, if flexion limit of screw-home motion is $\varphi_1 = 20\,°$ and the starting flexion limit of the free motion is $\varphi_2 = 40\,°$.

4. I developed a general method to the determination of the limit of the screw-home motion and the starting flexion limit of the free motion. I calculated these angles of healthy human knee joint's motion using this method.

   I carried out experiments on ten cadaver knee joints. I approximated the measurement data with trilinear function. I calculated the standard deviation of the approximation in function of φ_1 and φ_2 section limits. I searched for the minimum of the approximation in function of φ_1 , φ_2. I calculated with this method the section limits of the healthy human knee joint's rotation-flexion function. The limit of screw-home motion with 95% accuracy is

$$\varphi_1 = \bar{\varphi}_1 \pm 2{,}5\, s_1 = 17{,}75 \pm 1{,}075\,° \,,$$

   while the starting flexion limit of free motion with 95% accuracy is

$$\varphi_2 = \bar{\varphi}_2 \pm 2{,}5\, s_2 = 42{,}28 \pm 4{,}8\,° \,.$$

   I proved with measurement results on the basis of these approximations that the limit of screw-home motion is $\varphi_1 = 20\,°$, while the starting flexion limit of free motion is $\varphi_2 = 40\,°$. The rounding is expedients because of anatomical differences and the numbers of samples.

5. CONCLUSION AND SUGGESTIONS

The reference function was formulated above in the theses. This can take a new approach into the research of the knee joint and generally the joints. As a result, the results of experiments of the researchers performed with a variety of different methods would be comparable. Moreover, this mathematical model can be used in other joints.

The objective function can be used in the prosthesis ratings and the general implantation method optimization. In this area, the SZIU Biomechanics Research Group are already underway researches, and a doctoral topic was won to research in this area. The defined reference function is also able to help the prosthesis development engineers to develop a better prosthesis. Movements guaranteed by these advanced prostheses developed with the use of this reference function are much closer to the movements of the human knee joint, as the current prostheses provide (M. Csizmadia et. al., 2014). The exact reference function should be determine after performing a sufficient number of experiments, and I suggest the use of this reference function in the development of knee prostheses.

The sliding-rolling movements of the healthy human knee joint can be analyzed with using this experimental apparatus, which is forward looking, because doctoral thesis were formulated about the sliding-rolling movements of the prostheses in the Research Group . Thus, prosthesis ratings in accordance with another aspect are possible performing further experiments and using the results of the sliding-rolling motion.

These results also provide opportunity to establish the relationship between the determined reference function and sliding-rolling motion determinable using the experimental apparatus designed by me. The mathematical model formulated by me can be applied universally. However, the specific values of the objective function may depend on the movement form and the coordinate system. Therefore, decoupling of the mathematical model from the coordinate system is the basis of further researches. Furthermore, proving the dependence or independence from the movement form can be a further research opportunity. Medical point of view, it may be important to develop the independence of the reference function from the anatomical coordinate system. This may be relevant question- in terms of prosthesis implantation.

6. SUMMARY

The knee joint is one of the largest and most complex joint of the human body. Accordingly, its motion can be extremely complex. My goal was to establish a method based on experimental studies, which is suitable for describing the healthy human knee joint movement. My dissertation was confined to the main motion parameters specifically, which are flexion and rotation.
An experimental apparatus was presented in the doctoral thesis which allows kinematical analysis of human cadaveric knee joints. The apparatus allows unconstrained moving with modelling the forces in the active muscles working in case of the analysed motion form. A further advantage of the apparatus is the mobility. It is also suitable for the analysis of real human knee sliding-rolling motion in MRI or CT devices. The method of the experiments performed with this apparatus and the determination of the anatomical coordinate systems were detailed.
The evaluation method of the measurements performed with this apparatus using the detailed measurement method was discussed in the thesis. On the one hand the method of determining the position of the anatomical coordinate system from the measured data was detailed, on the other hand the mathematical method for determining the kinematical parameters of the knee motions in the anatomical coordinate system from the measured data system was shown.
The results were laid down on the basis of the experiments performed on ten human cadaver knee joint. The transformation method of the most important diagram, the flexion-rotation diagram used to describe the knee joint motion was presented.
The limit of the screw-home motion was also determined based on the results of these measurements. There wasn't generally accepted value for the limit of the screw-home motion in the literature earlier.
I found that the flexion-rotation diagram which describes the motion of the human knee joint can be approximated by a trilinear function. I also found that the linear approximation provides sufficient accuracy. The limits of trilinear functions on the one hand the limit of the screw-home motion determined by me, on the other hand the flexion position, where the motion be considered fully free. The section limits have not only in mathematics, but also physical content, so the trilinear approximation is completely founded.
The trilinear approximation is significant because it is a reference function that describes the healthy human knee joint's rotation. This function can be considered as an expected flexion-rotation diagram of an ideal prosthesis.

7. MOST IMPORTANT PUBLICATIONS RELATED TO THE THESIS

Referred articles in foreign languages

1. Fekete, G., De Baets, P., Wahab, M.A., Csizmadia, B., **Katona**, **G**., Vanegas-Useche, L.V., Solanilla, J.A. (2012): Slidig-Rolling Ratio during Deep Squat with Regard to Different Knee Prostheses. *Acta Polytechnica Hungarica,* 9 (5), pp. 5-24. (IF: 0,588)

2. Bíró, I., M. Csizmadia, B., **Katona, G**. (2008): New approximation of kinematical analysis of human knee joint. *Bulletin of the Szent István University* (Gödöllő) 16/17, pp. 330-338.

3. **Katona**, **G**., M. Csizmadia, B., Andrónyi, K. (2014): Determination of reference function to knee prosthesis rating. *Biomechanica Hungarica,* 6 (1), pp. 293-301.

4. M. Csizmadia, B., Balassa, G.P., **Katona**, **G**. (2014): The first steps to the development of the knee prosthesis rating method. *Biomechanica Hungarica* 6 (1), pp. 39-45.

5. Bíró, I., M. Csizmadia, B., **Katona**, **G**. (2010): Sensitivity investigation of three-cylinder model of human knee joint. *Biomechanica Hungarica* 3 (1), pp. 33-42.

6. **Katona**, **G**., M. Csizmadia, B., Bíró, I., Andrónyi, K., Krakovits, G. (2010): Motion analysis of human cadaver knee-joints using anatomical coordinate system. *Biomechanica Hungarica* 3 (1), pp. 93-100.