Szent István University
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Theses of Ph. D. Dissertation

MATHEMATICAL METHODS IN MARKETING DECISION –MAKING PROCESS

by
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The Doctoral School

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CONTENTS

1. PRELIMINARIES, GOALS

1.1 The Significance of the Chosen Topic

My Ph.D. dissertation deals with marketing decisions and the mathematical modelling of the decision-making process. People's opinions about marketing models can be grouped into two schools. One emphasises the importance of managers able to make marketing decisions based on intuition. The other group believes that marketing problems can be best approached using computerized mathematical methods.

Marketing is a dynamic, fast changing field where the duration of a given change is often shorter than the time taken to study it. Since marketing processes are continuously changing, a balanced position can rarely be reached. Because of the great number of relevant variables, it is difficult to determine the precise sources of change. Since marketing problems have behavioural and creative dimensions, they are typically poorly reproduced in forms allowing academic or analytical analysis. People who make decisions on an intuitive basis, think that the mathematical modelling of marketing processes is merely a waste of money and time. But, in fact, decisions based on intuition are only effective if the manager has already had experiences with the given topic in question. Before making a decision he looks for similar points within his own experience and then thinks through any contradictions encountered. In that way, the decision maker creates a kind of model based on his own "knowledge". Unlike the intuitive decision-makers, most of those who use mathematical models are forced to involve several professionals in the decision-making process.

The model presented in this paper may help to take into account a wide range of possibilities and alternatives, as well as to predict the consequences of decisions. The decision-maker may choose to disregard the results of the model if special considerations arise which cannot be placed within the framework of the model. However, if the model well presents the decision-making process of an organization, then any deviation from the model puts the decision-making person's consistency to the test and may put him in a position where he must explain the deviation.

1.2. Research Preliminaries

I first came into contact with the topic of marketing decision-making within my own work. The second-year students of GTK study several mathematical models within their Mathematical Programming II. subject , which are - either directly or else in modified form - suitable to the analysis of marketing processes. Such models are network-planning, dynamic programming, Markovian Chains or linear programming on a basic level.

In 2001, I submitted a research project proposal on this subject. My proposal was accepted by the Ministry of Agriculture, who financed my project. The research report was submitted in the required form prior to deadline and deemed a success by the Ministry.

1.3. Goals

I established my goals on the basis of the above- mentioned preliminaries. They can be summarized as follows:

1. My primary goal was the comparative analysis of mathematical methods and relations, with special regard to those stochastic process models with which timelines or their components can be examined.
2. I wish to develop a mathematical model which is suitable for recording stochastic processes using deterministic tools. I would like to reveal the potential of Markovian Chains through the innovative use of analytical approaches.
3. I wish to test the applicability of the resulting mathematical model. For this purpose I have chosen model products from two areas of agriculture: the world grain crop and the world mustard seed crop. Within the grain crop I will analyze the product group of wheat and barley, as marketshare fluctuations occur frequently amongst countries producing these crops. In the case of mustard seed, the group of producers is small, but for this reason the importance of each producer in the market is significant. The use of two highly different agricultural products for analytical purposes reveals the proposed model's precision and pertinence for use in diverse marketing decision making processes.

2. SUBSTANCE AND METHOD

The stochastic processes in which the consecutive states of a process depend on its previous states, are called Markovian Processes. If the parameter is time then only through the present can the future be influenced. In practise, this means that the output of certain experiments depends to a degree on the results of previous experiments. If the experiment is of the Markovian- type, we talk about discrete parameter-field Markovian Process or a Markovian Chain.

The $\{X_n\}$ probability series of $(n = 1,2,\ldots)$ is called a homogeneous Markovian Chain if the transitional probabilities are independent of *n*. To indicate the one-step transitional probabilities of the homogeneous Markovian Chain, let us use the following:

$$P_{jk} = P\{X_{n+1} = k | X_n = j\} \qquad (k, j = 0,1,2,\ldots)$$

If we place the P_{kj} numbers into a matrix, then we obtain:

$$P_{jk} = M = \begin{bmatrix} P_{00} & P_{01} & . & . & . & P_{0k} \\ P_{10} & P_{11} & & & & P_{ik} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ P_{j0} & P_{j1} & . & . & . & P_{jk} \end{bmatrix}$$

This is the one-step dimension – probability matrix of the homogeneous Markovian Chain. Based on the theory of multiplying probabilities, the (P_{jk}) the homogeneous Markovian Chain transitional probability matrix can only be stochastic. From the initial state we can calculate the probability of occurrence of the later states based on the matrix of *M*'s transitional probability. If the Markovian Chain is homogeneous, then the matrix of the one-step transitional probability consists of $P_{jk}(n)$ probability components. Each component is non-negative and is between 0 and 1 and the sum of the components in each line is 1 because

$$\sum_{k=0}^{\infty} P_{jk}(n) = \sum_{k=0}^{\infty} P\{X_{n+r} = k | X_n = j\} = 1$$

The sum of the components in the arbitrary line j is the probability of the result that the Markovian Chain goes through a j^{th} state, including the j^{th} state.
If the components of a quadratic matrix are not negative numbers and the sum of the components in each line equals 1, then the matrix is called a stochastic matrix.

The transitional probabilities can be calculated in the following way: n_{jt} is the number of objects in class j^{th} in time t^{th}, m_{jt} is the relative frequency. To the consecutive times we apply transitional probability matrix, the components of which are as follows:

$$P_{ijt} = \frac{n_{ijt}}{\sum_{j=1}^{r} n_{ijt}},$$

where n_{ijt} is the number of elements which were in the class of i^{th} in minus 1 time and crossed over to the class of j^{th} at t^{th} time. Thus we obtain a matrix series. The components of the matrix inform us about the probability of either moving into another class or staying in one place. The matrix series also inform us about the changes of these probabilities over time. It is also possible to follow the moving of the objects between classes. The matrix series can be characterised by the stability index as well:

$$C_{1t} = \frac{\sum_{j=1}^{r} p_{iit}}{r}$$

If C_{1t} is 1 or close to 1, than the object stays in the class which it reached. If the examined value of the object grows within the examined period of time, then it moves into a higher class. The value of C_{2t} then approaches 1.

If we know the relative frequency of this belonging to certain classes- the distribution within certain periods – then the matrix of the homogeneous transitional probability can be estimated using the Markovian equation.

$$m_{jt} = \sum_{i=1}^{r} \hat{P}_{ij} m_{it-1} + v_{jt}$$

where $\hat{P}_{ij}$ is the estimated transitional probability and v_{jt} is the probability of deviation.

By minimalising the squared sum of the differences we get the following quadratic programming problem:

$$\sum_{j=1}^{r} \hat{P}_{ij} = 1 \qquad i = 1,2,\ldots,r$$

$$0 \leq \hat{P}_{ij} \leq 1 \qquad \begin{matrix} i = 1,2,\ldots,r \\ j = 1,2,\ldots,r \end{matrix}$$

$$m_{jt} = \sum_{i=1}^{r} \hat{P}_{ij} m_{it-1} + v_{jt} \qquad \begin{matrix} i, j = 1,2,\ldots,r \\ t = 1,2,\ldots,r \end{matrix}$$

$$\overline{\sum_{j=1}^{r} \sum_{t=1}^{T} v_{jt}^{2} \to \min}$$

The first two conditions ensure that the matrix is stochastic. By minimalising the sum of the absolute value of the deviation using linear programming, we can obtain transitional probabilities.

$$\sum_{j=1}^{r} \hat{P}_{ij} = 1 \qquad i = 1,2,\ldots,r$$

$$0 \leq \hat{P}_{ij} \leq 1 \qquad \begin{matrix} i = 1,2,\ldots,r \\ j = 1,2,\ldots,r \end{matrix}$$

$$m_{jt} = \sum_{i=1}^{r} \hat{P}_{ij} m_{it-1} + v_{jt} \qquad \begin{matrix} i, j = 1,2,\ldots,r \\ t = 1,2,\ldots,r \end{matrix}$$

$$\overline{\sum_{j=1}^{r} \sum_{t=1}^{T} \left(v_{jt}^{+} + v_{jt}^{-} \right) \to \min}$$

The probability value of m_{jt} relative frequency can be represented by q_{jt} as shown in the following equation:

$$\sum_{j=1}^{r} q_{jt} = 1 \qquad t = 1,2,\ldots,T \,.$$

From the above calculated model's transitional probability values we can derive the stochastic matrix representing the transformation process.

3. RESULTS

3.1. Conclusions Drawn from Mustard Seed Production Data

Comparing the forecasts made with the proposed prognosis, the following statements can be made:

- Using average changes we often receive future values which go in the opposite direction from the dotted line. Especially in the case of those countries where the direction of the changes in data during the examined period is in consistent, on the basis of average changes, we can discount the first and last value correlation, since it is easy to predict that they will not materialize as consequences.
- The high degree polynomial trends integrated into wavy progressions precisely follow the course of the examined period; their relative rate of error remains under the acceptable level (15%). Because of their complexity, high degree and the relatively long timeline, the prognosis often shows unrealistic distributions for the future.
- Markovian-Chain model smooth the chronological wavings by their very nature. Of all the examined procedures, they offer the best way to follow the trend that we predicted while evaluating the data. The predictions resulting from this type of model always indicate that changes will be smaller and smaller in the future.

All of the four examined procedures predicted the market share of the countries in the production of mustard seed. In Chart 1, I present the market share prognosis, that is the summary of data-based predictions of market share.

Table 1. The Summary of Predictions for 2004

Country	*Average Absolute Change*	*Average Relative Change*	*Polynomial Trend*	*Markovian-Model*
Canada:	54,96%	54,96%	63,67%	55,85%
Nepal	28,51%	28,56%	0,68%	28,81%
Russia	8,95%	9,28%	9,06%	11,05%
USA	7,58%	8,24%	4,12%	4,29%
Sum:	**100%**	101,4%	77,53%	**100%**

The average absolute change and the Markovian model show a real distribution. The application of the average relative change is close to the required values, yet, the polynomial trend forecasts are far away from reality.

In this section, through non-forecast-related analysis, I reveal the stability of the examined process, and evaluate each chosen country's situation, as well as the potential for competitive disturbances, this using Markovian Chain-calculated transient probability matrix analysis. The modeling process included the 11 years from 1993 to 2003. I recorded the 10 transient probabilities, and calculated the model processes' transient probability matrix. Taking four countries into consideration, the matrix is a quadruple quadratic stochastic matrix.

Table 2.: The Matrix of the Transitional Probabilities in the Production of Mustard Seed

	Canada	**Nepal**	**Russia**	**USA**
Canada	0.64655	0.2139	0.11077	0.02878
Nepal	0.21559	0.60747	0.17694	0
Russia	1	0	0	0
USA	0.61474	0	0	0.38526

The analysis of the matrix of transitional probabilities has the direct result of providing us with important information. If we find numbers close to one by the main diagonal, it means that the production results of the countries in question, do not fall under that of other countries, so the production for the examined area can be considered constant for the longer term. Where this value is close to zero by the main diagonal, then the production results can be considered uncertain. Values by the main diagonal reveal the stability of the countries' production results. So Canada and Nepal are stable in the examined period. The share of these two countries in mustard seed production remains stable. The USA is in a more uncertain situation, and Russia can be considered insatiable due to its share of production results. We can establish that, among the four examined countries, the production role of the two countries having 72-88 % of the production can be considered constant.

The ratio of the sum of units on the main diagonal to the sum of all units in the table is called the stability indicator. When calculating the stability indicator of the mustard seed's production result for the examined period, one finds that the sum of the numbers in the main diagonal is 1.64 which is all the numbers in the stochastic matrix of the quadruple transitional probability, by definition 41.0 % of the value of 4. From this one can conclude that, in the examined period each country's share of the production results can be considered stable, in harmony with the concentrated productive role.

We can get a picture of the possibilities of the incidence of movement occurring during the timeline by comparing the probability of "climbing into a higher class" and the probability of "falling into a lower class". The indicator of "climbing into a higher class" can be derived from the quotient of the sum of the units above the main diagonal and the sum of the main diagonal. The indicator of "falling into a lower class" is determined by the quotient of the sum of the units below the main diagonal and the sum of the main diagonal. The sum of the values above the main diagonal is 0.53039. Divided by the sum of the main diagonal (1.64) we obtain a 32.4 % indicator of "climbing into a higher class". The sum of the values below the main diagonal is 1.6147, and the indicator value derived from this is 98.5 %. The higher class in this case refers to Canada and Nepal, the two bigger producers. The lower class is represented by Russia and the USA. We can conclude a kind of instability from the relative proportion of indicators. In the future the production rates will tend to move toward the countries in the lower class. This statement matches the prognosis of the USA's obtaining an increased share. (The USA represents the last row of the matrix.) If we can expect production rate regrouping of this nature, it could increase the value of the "falling into a lower class" indicator.

The individual rows of the transitional probability matrix tell us how likely it is that production results will move to other countries, and how likely it is that they will remain in place. The columns of the matrix inform us of the likelihood of each country obtaining an increased share of production from other countries.

Through the concrete examination of the meanings of each line, we can draw the following conclusions:

- Regarding Canada's line, we can state that the stability of its class is 64 %. When the country's share decreases within all agricultural products, then the likelihood of other countries increasing their production rate is as follows: Nepal 21 % probability of increase, Russia 11 % and the USA 3 %.

- There is 60 % chance of Nepal retaining its place, but if its share decreases, then all the other countries studied – with the exception of the USA – stand to increase their production rates by approximately 15-20%.
- There is 0% probability of Russia retaining its place. In the future we must count on some movement here. If its proportion decreases, only Canada's rate can increase.
- There is a 38 chance of the USA remaining stable, and only Canada stands to gain by a weakening in the USA's position.

By examining the meaning of each column we can draw the following conclusions:

- In terms of Canada's getting ahead, it reduces the share of any of the other three countries, certainly with different probabilities. With regards to Russia, this degree reaches the value of 1 or total probability.
- Nepal can only attain a higher rate at the disadvantage of the strongest country, Canada.
- Russia can decrease the shares of Nepal and Canada; within this Nepal has a 6% bigger probability of decreasing than does Canada.
- The USA can only be stronger if Canada's rate decreases.

3.2. Conclusions Drawn from World Grain Production Data

The Markovian Chain model can also be used for forecasting. The procedure employs stochastic methods, so its aim cannot be to exclude the effects of chance. Indeed, the advantage of this model and process is that one of its functions is to minimalize the effects of chance via the process of optimization, thus reducing the possible divergences from the minimum level, all within the given limits.

The way of calculating as follows:

- With the help of existing distribution data for past years, I retroactively calculated the production predictions. I thus obtained theoretical distributions called "ex-post" predictions.

- When I represent the group of countries' shares in the world's grain production using a common system of co-ordinates, the deviation of the calculations of the prediction from the real data can be easily traced.
- We can see how big the difference is by comparing the following: "ex-post" predictions made for the cross-over from each period to the next, using the following year's real distribution data; and calculationsmade for each year – where the exclusivity of the original year's data is ensured – using distribution values for the previous year.

Naturally, I made the calculations for every country or group of countries according to the nature of the model, but I will only represent the big producers here, as well as those countries which can influence Hungary's market share.

I show how much the choice of the period based on the "ex-post" predictions influences the use of the transitional probability matrix. The first producer examined from this point of view is Hungary.
The low values of the first chart's diagram show the results of the calculations in which I made the "ex-post" prediction on a permanent basis by using the year 1996. Thus every year was based on prediction and contained the values of minimized mistakes. The other line represents the calculations made using real distribution data for every year's "ex-post" prediction. I consider this procedure to be a more realistic approach when making a prognosis.

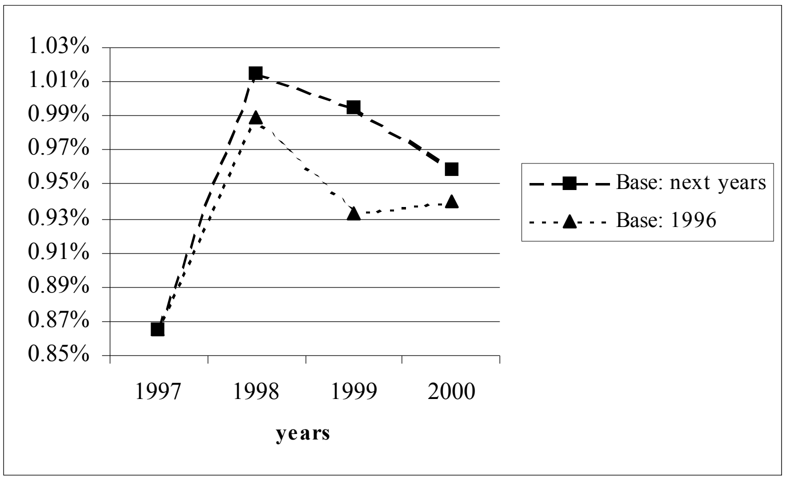

Chart 1.: Hungary's Sharing of the World's Grain Crop Production

In Chart 1 we can see that the two calculation methods do not show significantly different representations of the production in Hungary. It is important to note that, towards the end of the years considered, the distribution data obtained via the two processes approach one another in value. Examination of the value lines for the two prognoses reveals that the difference between them is quite insignificant, matching them 95% reliable. Let us now examine another instance of the difference between production value lines, this time for a more influential group of countries: the EU member nations.

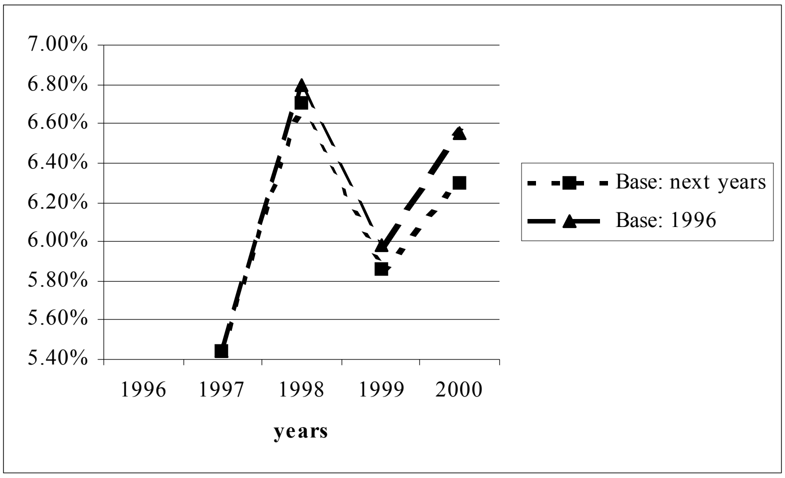

Chart 2.: EU's Sharing in the World's Grain Crop Production

By comparing the two kinds of "ex-post" predictions we can state that there is no significant difference between the methods. Even after four years of trials, the application of these models using "predicted data" shows mere a 0.3% deviation from the real data. If I compare the results with those obtained through statistical methods, we can state that the two data lines can be paired with "*t*" probation of two samples. Besides a 5 % error probability level, it does not reveal significant, statistically proven deviation.

The accuracy of prediction can also be observed by examining the degree of difference between Markovian Chain-based predictions and the real distribution values during the years in question. It is worth examining whether the differences between the prediction and the real distribution data remain within the 95 % reliability level.

Let's examine how big the difference is between the real data and the values of prognosis made by the known values in case of Hungary. The significant decrease found in 1997 in the case of Hungary can only be seen in chart 3 in delay in 1999, in the distribution data of the Markovian Chain. The last recorded year shows the same or close to equal value. There is no big difference between the real data and the predicted values in the graph. In the examined period the deviation of the values of the Markovian Chain prognosis from the real data is not significant; the error margin is approximately 5%.

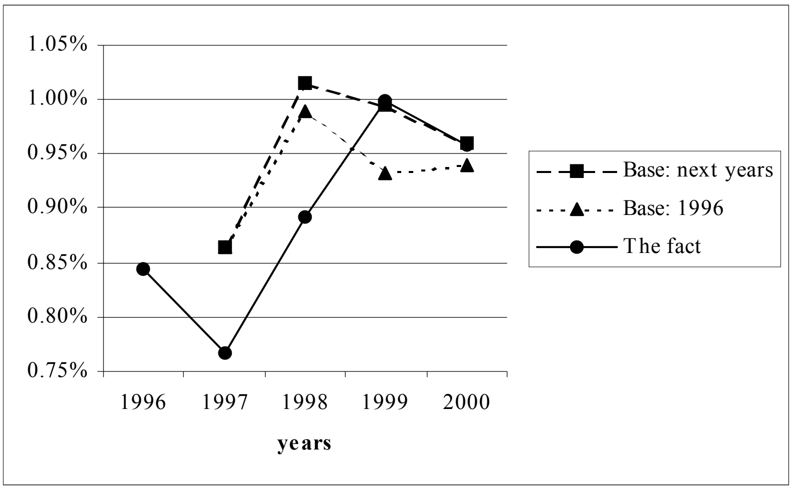

Chart 3.: Hungary's Share of the World Grain Crop Production

Now, let us examine the prediction values for a longer time period.

The calculations prove the soundness of the procedure in which I counted the data of *n+1* year on the basis of the real values of year *n*. The following procedure will be a hybrid type, because, in order to make calculations for further periods, only the method of basis-based prediction can be used for the unknown years.

Based on the earlier analyses I can claim that although the values obtained using the two types of calculation procedure differ somewhat, their results remain within the 95 % confidence level.

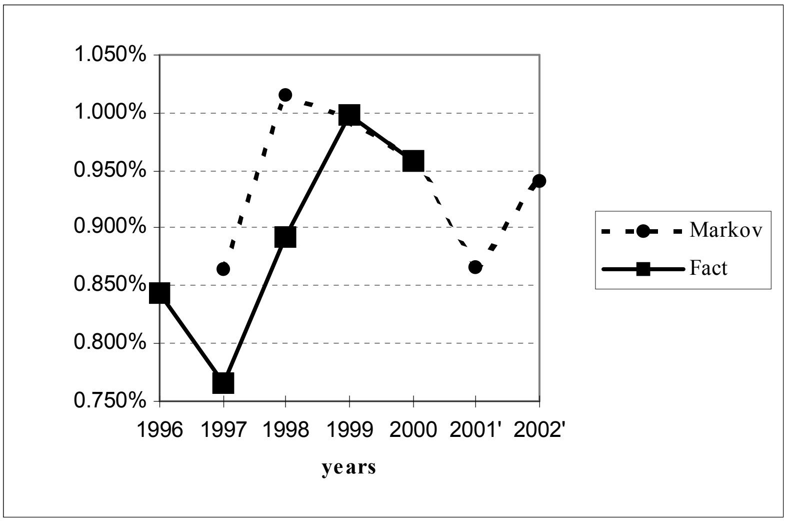

Chart 4.: Hungary's Real Share of World Grain Crop Production and the Markovian Chain Prognosis

Finishing the analysis of the Markovian Chains' temporary probabilities, we reach chart 4, which shows the formation of Hungary's production. Beside the real data we can see the values of the "ex-post" prognosis for the examined years. After reaching the end of the examined data line, the Markovian Chain data shows the probable proportion of the production's formation. According to the prediction, the role of Hungary's production will not decrease below 0.85 % in the predicted years, and will not increase above 0.95 %. According to the values in the last examined year the production rate decreases and then increases again within the 0.1 % value zone. The prognosis includes the memories of changes in the earlier years. The run of the prognosis almost repeats the change between 1996 and 1998, as well as that of higher proportion levels.

3.3. New Academic Results

1. I analysed and evaluated those mathematical models used in marketing decision-making, which are mainly suitable for comparing timelines. I made the applicability conditions of the examined methods more precise by applying findings documented in pertinent technical literature.
2. Through my research, I developed a mathematical model based on Markovian Chain analysis which, although the connections with many of its components are well-known, has not been used in published research for integration in timeline-related analysis.
3. Having tested the proposed mathematical model, I can state that the predictions made by this new method are more exact than the ones made by the usual statistical methods.
4. As a result of the calculations for the model products, I can say that the conclusions which can be drawn regarding the market position or the trading of a certain product could only be achieved using other methods by investing more time or money.

4. CONSEQUENCES AND SUGGESTIONS

My suggestions concerning the practical applications of my research are as follows:

- The methods described in my paper are suitable for modeling the market effects of marketing processes
- The models described in the technical literature can be analysed effectively using the multivariable or stochastic methods in some parts of marketing.
- The division of products and processes can be modeled with the Markovian Chain method.
- The Markov Chain method allows the establishment of transitional probability stochastic matrixes, the analysis of which provides important information to producers about the present and the future role in the market place.
- Using Markovian Chain forecasting, it is possible to make predictions about market share, which ‘remember’ the fluctuation of earlier periods and project it into the future
- The predictions made in this way complement mathematical statistics models which are used to predict other aspects of marketing.

5. PERTINENT PUBLICATIONS

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VIII nemzetközi Agrárökonómiai tudományos napok. 2002. ISBN 963 9256 75 7. p. 212.

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XXIX Óvári tudományos Napok 2002. ISSN 0237-9902 p. 220.

Ugrósdy György
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Studies in Agricultural Economics. AKII 2003. Budapest (forthcoming)

Ugrósdy György
"A világ búzatermelésének vizsgálata."
Gazdálkodás 2002. (2.) p. 67-71.